Energy Storage

2. Introduction to GAMS

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Operational Research

- Operational Research is a discipline that deals with the application of advanced analytical methods to help make better decisions
- Modelling of deterministic and probabilistic systems from reality
- It is used in the management of:
  - Economy at the state level
  - Corporate business
  - Economy
  - Natural sciences
  - Sociological sciences
  - Engineering

Linear programming

- LP solves problems of allocating limited resources among the activities within the system in an optimal manner
- It is used in any number of situations such as:
  - Allocation of production plants
  - Production scheduling
  - Various kinds of games
  - ….

Linear programming

- The adjective *linear* means that all mathematical functions within the model must be linear
- *Programming* is a synonym for planning and does not apply to computer programming
- Planning activities in order to achieve the objective in the best of all possible ways

Example

- The company produces two types of windows:
  - Window with a PVC frame
  - Windows with a wooden frame
- The company can sell all produced windows
- Products are assembled in three factories
- Each factory has certain production capacity

Example

- How many products of each type should be produced by the company to achieve maximum profit?

Factory	The required capacity of the plant for product 1 (PVC window)	The required capacity of the plant for product 2 (wooden window)	Available capacity
1	1	0	4
2	0	2	12
3	3	2	18
Profit per unit	3 $	5 $	

Example

- LP problem formulation:
  - x_1 i x_2 – number of manufactured products of each type
  - Z – profit per time unit
- The first company can produce at most 4 PVC windows in a single time unit:

$$x_1 \leq 4$$

- The second company can produce at most 6 wooden windows in a single time unit:

$$2x_2 \leq 12$$

- The third company can produce at most 6 PVC windows or 9 wooden windows in a single time unit:

$$3x_1 + 2x_2 \leq 18$$

Example

- Since the production can not be negative the following constraints are imposed:

$$x_1 \geq 0 \qquad x_2 \geq 0$$

- Problem formulation:

$$\text{Maksimizirati} \quad Z = 3x_1 + 5x_2$$

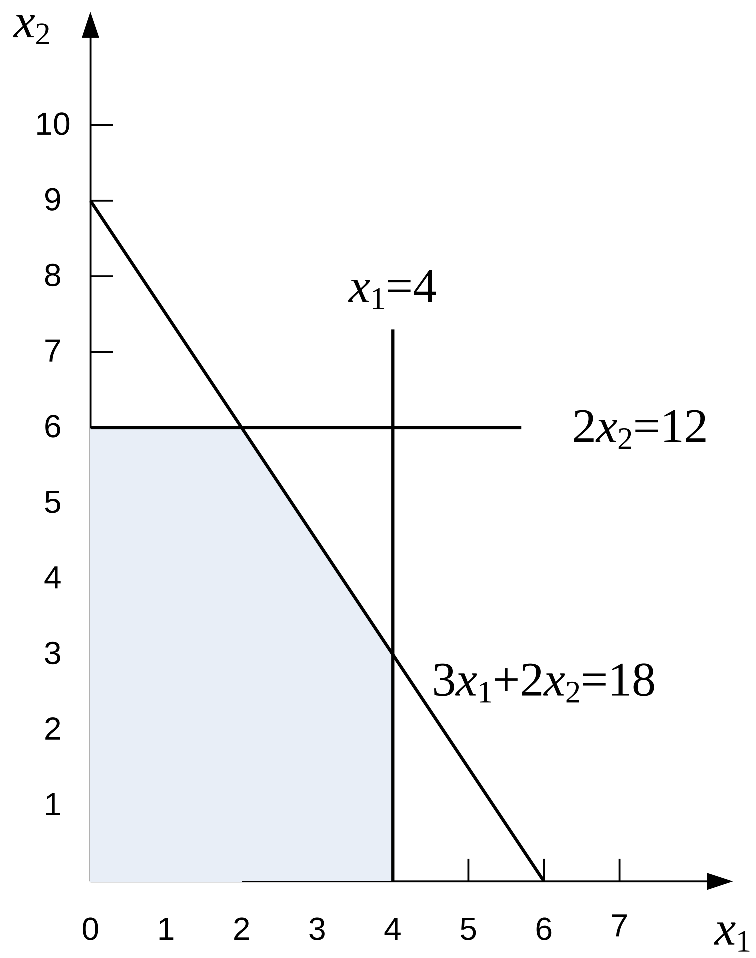
- Constraints:

$$\begin{aligned} x_1 \qquad\qquad &\leq 4 \\ 2x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 18 \end{aligned}$$

$$x_1 \geq 0 \qquad x_2 \geq 0$$

Example

- Graphical solution:



GAMS

- GAMS (*General Algebraic Modeling System*) is a computer environment that can easily be used to model and solve optimization problems
- GAMS has capability to solve from small scale to large problems with small code by means of the use of index to write blocks of similar constraints from only one constraint
- The model is independent from the solution method, and it can be solved by different solutions methods by only changing solver

GAMS features

- A GAMS model is a collection of statements in in the GAMS language
- Every GAMS statement should be terminated with a semicolon
- GAMS compiler does not distinguish between upper-and lower letters
- There are two ways to insert documentation within a GAMS model:
  - any line that starts with an asterisk (*) in column 1 is disregarded as a comment line by the GAMS compiler
  - documentary text can be inserted within specific GAMS statements ($ontext/$offtext)

GAMS features

- The creation of GAMS entities involves two steps: a declaration and an assignment or definition
  - declaration means declaring the existence of something and giving it a name
  - assignment or definition means giving something a specific value or form
  - in case of equations, declaration and definition are made in separate GAMS statements
  - for all other GAMS entities there is an option of making declarations and assignments in the same statement or separately
- The names given to the entities of the model must start with a letter and can be followed by up to thirty more letters or digits

Parts of a GAMS Model

COMMAND	PURPOSE
SET	Used to indicate the name of the indices
SCALAR	Used to declare scalars
PARAMETER	Used to declare vectors
TABLE	Used to declare 2D vectors
VARIABLE	Declare variables, assign type and bounds
EQUATION	Function to be optimized and its constraints
MODEL	Give a name to the model, and list the included constraints
SOLVE	Indicates which solver to use
DISPLAY	Indicates what will be displayed in output

1. Sets

- Sets define the length of the variables
- Sets represent the index in algebraic expressions
- Sets examples:

```
set i /1*7/;
set i = {1,2,3,4,5,6,7};
set t /t1*t52/;
```

2. Parameters

- Parameters are used for data entry that will not change
- Example:

```
set i /1*7/;
parameter numberofletters(i)    /1 10
                                 2  6
                                 3  7
                                 4  8
                                 5  5
                                 6  6
                                 7  7/
parameter doublenumber(i)=2*numberofletters(i);
parameter
onlyweekend('6','7')=3*numberofletters('6','7');
```

3. Variables

- Variables value is not known in advance and can be changed
- Variables types:
  - Variable $-\infty$ do $+\infty$
  - Positive variable 0 do $+\infty$
  - Binary variable 0 ili 1
  - Integer variable 0, 1, ..., 100

4. Equations

- The main difference with C programming language
- It is possible to create a large number of equations at once
- Equations can not be declared and defined in the same row
- Relational operators:

=l=	less than or equal to
=g=	greater than or equal to
=e=	equal to

5. Model

- The model represents a set of equations
- The model must have a title and must be declared
- Model title is always followed by a list of equations that are included in the model
- Example:

```
model myfirstmodel /all/;
```

6. Solve

- Solve is the command used to solve the optimization problem that we defined:
  - lp – linear programming
  - nlp – non linear programming
  - mip – mixed integer programming
- As a solution of optimization problem solver provides the optimized variable
- Example:

```
solve myfirstmodel using lp minimizing z;
```

Solve Example 1 in GAMS

- Determine the energy costs of a building whose hourly consumption is given in table (in kW):

Hour	Demand	Hour	Demand	Hour	Demand
1	100	9	380	17	550
2	120	10	400	18	480
3	140	11	450	19	530
4	110	12	520	20	490
5	120	13	540	21	440
6	150	14	520	22	460
7	240	15	500	23	380
8	330	16	520	24	240

Solve Example 1 in GAMS

- Energy price is 0.90/0.45 kn during the high/low tariff
- High tariff is active from 10 pm until 7 am
- Allowed peak power is 500 kW
- Battery storage has the following features:
  - Rated power 100 kW,
  - Capacity 350 kWh,
  - Charging efficiency 0.88
  - Discharging efficiency 0.95

Example 1 – Formulation

$$\text{Minimize} \quad \sum_t C_t \cdot \Delta^{\mathrm{T}} \cdot \hat{d}_t \tag{1}$$

subject to

$$\hat{d}_t = D_t + p_t^{\mathrm{ch}} / \eta^{\mathrm{ch}} - p_t^{\mathrm{dis}} \cdot \eta^{\mathrm{dis}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{2}$$

$$s_t = s_{t-1} + p_t^{\mathrm{ch}} \cdot \Delta^{\mathrm{T}} - p_t^{\mathrm{dis}} \cdot \Delta^{\mathrm{T}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{3}$$

$$s_t \leq SOC^{\mathrm{max}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{4}$$

$$p_{t,}^{\mathrm{ch}} \leq P^{\mathrm{bat}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{5}$$

$$p_t^{\mathrm{dis}} \leq P^{\mathrm{bat}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{6}$$

$$\hat{d}_t \leq P^{\mathrm{distr}}, \quad \forall t \in \Omega^{\mathrm{T}} \tag{7}$$

$$p_t^{\mathrm{ch}}, p_t^{\mathrm{dis}}, s_t \geq 0, \quad \forall t \in \Omega^{\mathrm{T}} \tag{8}$$

Example 1 – Parameters

Parameters

C_t	Cost of electricity, kn/kWh.
D_t	Overall building consumption at hour t, kW.
P^{bat}	Battery (dis)charging capacity, kW.
P^{distr}	Maximum allowed building load, kW.
SOC^{max}	Energy capacity of the battery, kWh.
Δ^{T}	Time interval length (1 h).
η^{ch}	Battery charging efficiency.
η^{dis}	Battery discharging efficiency.

Example 1 – Variables

Variables

$\hat{d}_t$ Power from the grid at hour t, kW.

p_t^{ch} Battery charging at hour t, kW.

p_t^{dis} Battery discharging at hour t, kW.

s_t Battery state of charge at the end of hour t, kWh.

Example 2 – Optimal Microgrid Operation

- The expected non-dispatchable loading during 6 hour period is given in the table. The microgrid also comprises of two generating units and a controllable load. The first generating unit has rated power 150 kW and operating costs 40 €/MWh, while the second unit has rated power 150 kW and operating costs 60 €/MWh. Controllable load has rated power 25 kW. Determine the operation costs in the following cases:
  1. Controllable load cannot be controlled.
  2. Controllable load is lighting (0% retrieveal factor) and it is possible to shift 20% of consumption in each hour, a total of 12.5 kWh during 6 hours.
  3. Controllable load is HVAC (120% retrieval factor) and it is possible to shift 20% of consumption in each hour, a total of 12.5 kWh during 6 hours.
  4. Controllable load cannot be controlled, but a battery is included in the microgrid; charging and discarding efficiencies are 0.9 (initial condition – battery is empty).
  5. Combination of controllable load from 3 and battery from 4.

Hour	Load
1	80 kW
2	160 kW
3	220 kW
4	90 kW
5	200 kW
6	180 kW